Leading MATH Success

TIPS4RM

TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

Continuum and Connections

Solving Equations and Using Variables as Placeholders

Overview

Context Connections

- Positions equations in a larger context and shows connections to everyday situations, careers, and tasks
- Identifies relevant manipulatives, technology, and web-based resources for addressing the mathematical theme

Connections Across the Grades

- Outlines the scope and sequence using Grade 6, Grade 7, Grade 8, Grade 9 Applied and Academic, and Grade 10 Applied as organizers
- Includes relevant specific expectations for each grade
- Summarizes prior and future learning

Instruction Connections

- Suggests instructional strategies, with examples, for each of Grade 7, Grade 8, Grade 9 Applied, and Grade 10 Applied
- Includes advice on how to help students develop understanding

Connections Across Strands

- Provides a sampling of connections that can be made across strands, using the theme (equations) as an organizer

Developing Proficiency

- Provides questions related to specific expectations for a specific grade/course
- Focuses on specific knowledge, understandings, and skills, as well as on the mathematical processes of Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, and Connecting. *Communicating is part of each question.*
- Presents short-answer questions that are procedural in nature, or identifies the question as problem solving, involving other mathematical processes, as indicated
- Serves as a model for developing other questions relevant to the learning expected for the grade/course

Problem Solving Across the Grades

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Models a variety of representations and strategies that students may use to solve the problem and that teachers should validate
- Focuses on problem-solving strategies, involving multiple mathematical processes
- Provides an opportunity for students to engage with the problem at many levels
- Provides problems appropriate for students in Grades 7–10. The solutions illustrate that the strategies and knowledge students use may change as they mature and learn more content.

Is This Always True?

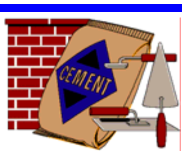
- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Focuses on the mathematical processes Reasoning and Proving, and Reflecting

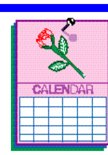
Solving Equations and Using Variables as Placeholders

Context

- What is a variable? What does an equal sign mean? What is an expression? What is an equation? Clear understanding of the answers to these questions will help students develop the algebraic thinking that leads to the study of functions and relations in secondary school.
- Variables are used for placeholding, making rules, or for generalizing. Particular attention must be paid to the development of these concepts to help students develop algebraic fluency.
- Students are introduced in early grades to the concept of a "placeholder" variable in questions using the form $\square + 3 = 8$ and $2 \times \square = 16$. This leads to equations using the form $a + 3 = 8$ and $2n = 16$.
- In Grades 7, 8, 9, and 10 students create and solve a variety of equations that have one specific solution, thereby adding the creation and solving of equations to their problem-solving strategies.
- Students also use variables to create equations that define rules for relationships (e.g., $a = 2b + 6$ or $C = 2\pi r$) and equations that state generalizations (e.g., $a + b = b + a$, where there is no relationship between the variables).
- Students extend their thinking about the equal sign from "get the answer" to being an essential part of a mathematical statement that shows equality between two expressions.
- When known values are substituted into a formula to find an unknown value, the connection is made between the rulemaking equation and the equation to be solved for the unknown placeholder in a specific case.

Context Connections


Ratios


Calendars


Crop Yields


Electricity Formulas

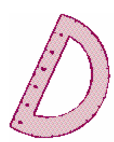

Speed/Distance/Time

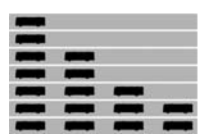

Exchange Rates


Measurement Formulas


Time Zones


Wages


Angle Relationships


Spreadsheets


Other Connections

Manipulatives
- tiles
- balance
- cubes

Technology
- spreadsheets
- The Geometer's Sketchpad® 4
- calculators/graphing calculators

Other Resources
www.standards.nctm.org/document/chapter3/alg.htm#bp2
http://www.purplemath.com/modules/variable.htm
http://math.rice.edu/~lanius/Lessons/calen.html
http://www.uni-klu.ac.at/~gossimit/pap/guest/misconvar.html
http://www.uwinnipeg.ca/~jameis/New%20Pages/MYR39.html

Connections Across Grades

Selected results of word search using the *Ontario Curriculum Unit Planner*

Search Words: *equation, formula, variable, algebraic*

Grade 6	Grade 7	Grade 8	Grade 9	Grade 10
▪ develop the formulas for the area of a parallelogram and the area of a triangle, using the area relationships among rectangles, parallelograms, and triangles; ▪ determine, through investigation using a variety of tools and strategies, the relationship between the height, the area of the base, and the volume of a triangular prism, and generalize to develop the formula; ▪ use variables in simple algebraic expressions and equations to describe relationships; ▪ demonstrate an understanding of different ways in which variables are used; ▪ identify, through investigation, the quantities in an equation that vary and those that remain constant; ▪ solve problems that use two or three symbols or letters as variables to represent different unknown quantities; ▪ determine the solution to a simple equation with one variable, through investigation using a variety of tools and strategies.	▪ develop and represent the general term of a linear growing pattern, using algebraic expressions involving one operation; ▪ compare pattern rules that generate a pattern by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term with pattern rules that use the term number to describe the general term; ▪ model everyday relationships involving constant rates, using algebraic equations with variables to represent the changing quantities in the relationship; ▪ translate phrases describing simple mathematical relationships into algebraic expressions, using concrete materials; ▪ evaluate algebraic expressions by substituting natural numbers for the variables; ▪ make connections between evaluating algebraic expressions and determining the term in a pattern using the general term; ▪ solve linear equations of the form $ax = c$ or $c = ax$ and $ax + b = c$ or variations such as $b + ax = c$ and $c = bx + a$ (where a, b, and c are natural numbers) by modelling with concrete materials, by inspection, or by guess and check, with and without the aid of a calculator.	▪ determine a term, given its term number, in a linear pattern that is represented by a graph or an algebraic equation; ▪ describe different ways in which algebra can be used in everyday situations; ▪ translate statements describing mathematical relationships into algebraic expressions and equations; ▪ evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables; ▪ make connections between solving equations and determining the term number in a pattern, using the general term; ▪ solve and verify linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a "balance" model.	**Applied and Academic** ▪ relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations; ▪ describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three; ▪ substitute into and evaluate algebraic expressions involving exponents; ▪ pose problems, identify variables, and formulate hypotheses associated with relationships between two variables. **Applied** ▪ add and subtract polynomials involving the same variable up to degree three; ▪ multiply a polynomial by a monomial involving the same variable to give results up to degree three, using a variety of tools; ▪ solve first-degree equations with nonfractional coefficients, using a variety of tools and strategies; ▪ substitute into algebraic equations and solve for one variable in the first degree. **Academic** ▪ add and subtract polynomials with up to two variables; ▪ multiply a polynomial by a monomial involving the same variable; ▪ expand and simplify polynomial expressions involving one variable; ▪ solve first degree equations including equations with fractional coefficients using a variety of tools; ▪ rearrange formulas involving variables in the first degree, with and without substitution; ▪ solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods.	**Applied** ▪ solve first-degree equations involving one variable, including equations with fractional coefficients; ▪ determine the value of a variable in the first degree, using a formula; ▪ express the equation of a line in the form $y = mx + b$, given the form $ax + by + c = 0$; ▪ manipulate algebraic expressions, as needed to understand quadratic relations; ▪ solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination; ▪ solve problems that arise from realistic situations described in words or represented by given linear systems of two variables, by choosing an appropriate algebraic or graphical method.

Summary of Prior Learning

In earlier years, students:

- solve simple equations with one variable using a variety of tools and strategies (including concrete materials, guess and check);
- develop the concept that equations consist of variables (unknown quantities that vary) and constants;
- represent equations using variables and symbols (e.g., $5 + \square = 12$, $5 + s = 12$);
- understand inverse relationships (addition and subtraction; multiplication and division).

In Grade 7, students:

- use equations to model real-life situations;
- solve linear equations in the form $ax = c$, $c = ax$, $ax + b = c$, $c = bx + a$, using a variety of methods;
- use equations in measurement formulas to find area of trapezoids, area of composite figures, and volume of right prisms.

In Grade 8, students:

- use equations to generalize patterns;
- solve simple linear equations using inspection, guess and check, and balance model;
- use equations in measurement formulas to solve problems involving circumference and area of two-dimensional shapes and involving surface area and volume of cylinders;
- use equations in solving problems involving angle relationships.

In Grade 9 Applied, students:

- solve first-degree equations in one variable with non-fractional coefficients;
- use equations to solve problems in proportional reasoning;
- use equations to solve problems involving the Pythagorean theorem;
- use equations to solve problems involving measurement formulas of perimeter and area of composite figures, volumes of three-dimensional solids, and in determining geometric relationships;
- connect equations to other representations and use to model and solve problems.

In Grade 10 Applied, students:

- solve first-degree equations in one variable with fractional coefficients and rearrange equations and formulas by isolating a variable;
- apply equation solving in real-life problems involving similar triangles and trigonometry of right triangles;
- use equations in solving problems involving surface area and volume;
- use equations to graph and write lines in the form $y = mx + b$;
- extend equation solving skills to solve and interpret systems of linear equations, using graphical and algebraic methods;
- use second-degree equations to represent quadratic relations and solve problems graphically;
- connect the forms of the quadratic relation $y = x^2 + bx + c$ and $y = (x – r)(x – s)$ graphically and algebraically.

In later years

Students' choice of courses will determine the degree to which they apply their understanding of concepts related to solving equations and using variables as placeholders.

Instruction Connections

Suggested Instructional Strategies

Grade 7

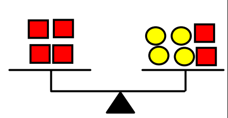
- Have students explain in words the meaning of simple expressions in context, e.g., $25q$ cents represents the value of q quarters, or $10n$ dollars represents the payment for n hours of work at \$10/h.
- Discuss the range of values that a variable could represent considering the context, e.g., Could it be zero? 1000? 3.5?
- Develop understanding that the variable in a first-degree expression is a symbol that may be replaced by a given set of numbers.
- Generate a table of values using an expression and use one possible value to form an equation.

n	1	2	3	4	5	6	7	8
$21n$	21	42	63	84	105	126	147	168

An equation created from the table of values is $21n = 105$.
The variable n in the equation is now a placeholder variable that has one unique value ($n = 5$).

- Generate a graph from a table of values. Connect the solution to an equation as the value of one variable which can be read from the graph.
- Connect to prior learning – students compare $21 \times \square = 105$ to $21n = 105$; students make the communication transition from “finding the number that goes in the box” to “solving the equation.”
- Solve simple first degree equations of the forms $ax = c$ and $ax + b = c$ with whole number solutions only by inspection and systematic trial – students give reasons for the values they choose to try.
- Guide students to use their knowledge from an incorrect guess to make a more educated subsequent guess, e.g., Is there a larger or smaller difference between the left and right sides of the equation for the current trial compared to the previous trial? Should you use a higher or lower value for the variable in your next trial?
- Discuss equivalent representations that yield the same solution (e.g., The length is 10 *m* which is 6 m longer than the width) and can be represented as $6 + w = 10$ or $10 - w = 6$ or $10 - 6 = w$ or $w + 6 = 10$ or $6 = 10 - w$.
- Ask students to explain in words the meaning of simple formulas, e.g., $A = bh$, $P = 2l + 2w$. Use area and perimeter formulas, e.g., If the area of a rectangle is 240 m^2 and its base is 60 m long, what equation could you use to determine the height of the rectangle? Answer: Since $A = bh$ is the formula for the area of a rectangle, then the equation is $240 = 60h$ or $240 = 60 \times h$.
- Develop understanding that a solution to an equation is a value that makes the left side equivalent to the right side, e.g., the two sides can be equivalent masses, equivalent dollar values, equivalent perimeters, etc. Students explain how they know that their solution makes the equation true.

Grade 8



- Begin activities that will lead to the use of formal algebraic solutions for solving equations.
- Use a balance and equivalent masses to establish that to preserve balance in an equation you must add the same quantity to both sides, or take the same quantity away from both sides, or divide both sides by the same number, etc.

Helping to Develop Understanding

- Explain the shorthand use of $3n$ for $3 \times n$. Compare this to $n + 3$ and $3 + n$; what is the same? what is different?
- Use different variables to help students realize that the equations $21n = 105$ and $21t = 105$ have the same solution – compare $21n = 105$ with $105 = 21n$.
- Distinguish between expressions and equations, e.g., $2x$ is an expression, $2x + 3 = 9$ is an equation. Formulas are equations.
- Provide frequent feedback on their mathematical form as students evaluate expressions and solve equations (= sign down the left as the expression becomes simpler and simpler versus = sign down the middle between expressions as one side of an equation simplifies to just the variable).
- A statement that identifies a variable needs to be very clear – “Let n represent the marbles” does not clearly state which attribute is represented – Is it the number of marbles? The radius of one marble? The total volume of all the marbles? Include units when appropriate.
- Contrast the instructions when students are to evaluate an expression, simplify an expression, and solve an equation.
- When students have a deeper understanding of using balance to solve equations, they will naturally eliminate some lines in an algebraic solution. Teaching “shortcuts” detracts from understanding. Sample solution:

$$4x - 3 = 7$$
$$4x - 3 + 3 = 7 + 3$$
$$4x = 10$$
$$\frac{4x}{4} = \frac{10}{4}$$
$$x = 2.5$$

- Check the solution to a linear equation and emphasize it is the only value that will make the left side equivalent to the right side.
- Use concrete materials to provide a visual representation of the balance method to solve equations with positive terms. As students become proficient, make transition to an algebraic approach.
- Integrate the use of computer algebra systems to consolidate understanding of algebraic method for solving equations.
- Emphasize the importance of verifying solutions by doing either a formal check or a mental check.
- Ask students to reflect on whether the solution is reasonable.
- Explore different methods of solving equations that have fractional coefficients.

Suggested Instructional Strategies	Helping to Develop Understanding
• Develop understanding that a first degree equation has a maximum of one solution. Students work on an equation that comes from a context and a set of "test" numbers – include the solution as well as numbers above and below the solution – and explain why they think the solution is unique. • Connect the finding of a term number in a pattern to solving an equation. • Use equation solving to solve for an unknown angle in a triangle given two of the angles. **Grade 9 Applied** • Create and solve more complex equations that have integer coefficients, not fractional coefficients. • Provide contextual problems as a basis for solving equations. Graph the relationship to provide a connection between linear relationships and solving equations. • Connect solving an equation to determine a missing coordinate of an ordered pair from the related graph. Allow this as a valid approach to solving equations but provide problems that make it increasingly difficult to rely on a graphical approach. • Use measurement formulas and geometry to provide a context for solving equations – students are required to substitute into formulas and solve for one variable that is not the subject of the formula. • Stress the importance of inverse operation (rather than opposite sign) when making the transition to the algebraic method for solving equations. • When solving for a side of a right-angled triangle using the Pythagorean theorem, stress the "opposite" or "inverse" operation of x^2 as the square root of x, thus the method for solving. This is consistent with students' previous understanding of solving equations. **Grade 10 Applied** • Solve for a variable in a formula and then substitute in a known value. Connections can be made to measurement formulas and formulas derived from linear relationships.	• Compare the features that distinguish an equation with brackets from an expression with brackets. • Solve systems of two linear equations in two variables, and connect to the graphical solution. • Ask students to reflect on which method is most appropriate when solving a given linear system.

Connections Across Strands

Note
Summary or synthesis of curriculum expectations is in plain font.
Verbatim curriculum expectations are in italics.

Grade 7

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• apply computational strategies to solve problems • use estimation when solving problems • evaluate expressions that involve whole numbers and decimals • demonstrate an understanding of adding and subtracting of integers	• develop skills with equations through measurement formulas (e.g., area of trapezoid, area of composite figures, volume of right prisms) • substitute into measurement formulas • develop the formula for the area of a trapezoid • solve problems involving the area of a trapezoid • develop the formula for the volume of right prisms with polygonal bases • *solve problems involving the volume and surface area of right prisms*	N/A	• see Connections Across Grades, p. 3	• draw conclusions from data • identify trends in data

Grade 8

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• use exponential notation • use estimation when solving problems • represent and solve problems involving operations with integers	• develop, through investigation, the formulas for circumference and area of a circle, and use to solve problems • determine the formula for the volume and surface area of a cylinder • solve problems involving the surface area and the volume of cylinders, using a variety of strategies	• use equations to determine geometric relationships, and in solving angle-relationship problems • determine the algebraic relationship between the number of faces, edges, and vertices of a polyhedron	• See Connections Across Grades, p. 3	• compare the theoretical and experimental probability of an event • interpret and draw conclusions from data • identify trends based on rate of change of data

Grade 9 Academic

Number Sense and Algebra	Measurement and Geometry	Analytic Geometry	Linear Relationships
• See Connections Across the Grade. p. 3.	• relate the geometric and algebraic representations of the Pythagorean theorem • apply the Pythagorean theorem to solve problems • *solve problems involving the areas and perimeters of composite two-dimensional shapes* • *develop, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere* • solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures • use equations to determine geometric relationships, and in solving problems	• distinguish between linear and non-linear relationships, given equations and relate linear relations to constant rate of change • write equations of lines in the form $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$ • *express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$.* • explain the geometric significance of m and b in the equation of a line, $y = mx + b$ • graph lines by hand, using a variety of techniques • determine the equation of a line given various information such as slope and y-intercept, slope and a point, two points • *describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation* • *identify and explain any restrictions on the variables in a linear relation arising from a realistic situation* • determine the point of intersection of two linear relations graphically and interpret its significance in context	• represent linear relations, from realistic situations, using a variety of representations, including equations • *determine the equation of a line of best fit for a scatter plot, using an informal process* • *determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation* • *determine other representations of a linear relation, given one representation* • *describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied*

Grade 9 Applied

Number Sense and Algebra	Measurement and Geometry	Linear Relations
• *solve for the unknown value in a proportion, using a variety of methods* • See Connections Across Grades, p. 3	• relate the geometric and algebraic representations of the Pythagorean theorem • apply the Pythagorean theorem to solve problems • use measurement formulas to solve problems involving the areas and perimeters of composite two-dimensional shapes • *develop, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere* • *solve problems involving the volumes of prisms, pyramids, cylinders, cones, and spheres* • use equations to determine geometric relationships, and in solving problems	• *determine, through investigation, connections among the representations of a constant rate of change of a linear relation* • *compare the properties of direct variation and partial variation in applications, and identify the initial value* • *express a linear relation as an equation in two variables, using the rate of change and the initial value* • *describe the meaning of the rate of change and the initial value for a linear relation arising from a realistic situation, and describe a situation that could be modelled by a given linear equation* • use and compare various representations (e.g., to determine values of a linear relation) and in solving problems • *describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied*

Grade 10 Applied

Measurement and Geometry	Modelling Linear Relations	Quadratic Relations of the Form $y = ax^2 + bx + c$
• solve problems involving similar triangles • solve problems involving the primary trigonometric ratios and the Pythagorean theorem • *solve problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving combinations of these figures, using the metric system or the imperial system, as appropriate*	• *write equations of lines in the form $y = mx + b$ and the special cases $x = a$, $y = b$* • *explain the geometric significance of m and b in the equation of a line, $y = mx + b$* • *express an equation of the form $y = mx + b$ given the form $Ax + By + C = 0$* • *graph lines by hand, using a variety of techniques* • *determine the equation of a line, given its graph, the slope and y-intercept, the slope and a point on the line, or two points on the line* • *determine graphically the point of intersection of two linear relations* • *solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination* • *solve problems that arise from realistic situations described in words or represented by given linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method*	• *determine, through investigation using technology, that a quadratic relation of the form $y = ax^2 + bx + c$ (**a** not equal to 0) can be graphically represented as a parabola, and determine that the table of values yields a constant second difference* • *compare, through investigation using technology, the graphical representations of a quadratic relation in the form $y = x^2 + bx + c$ and the same relation in the factored form $y = (x – r)(x – s)$, and describe the connections between each algebraic representation and the graph*

Developing Proficiency

Grade 7

Name:
Date:

Expectation – Patterning and Algebra, 7m68: Solve linear equations of the form $ax = c$ or $c = ax$ and $ax + b = c$ or variations such as $b + ax = c$ and $c = bx + a$ (where a, b, and c are natural numbers) by modelling with concrete materials, by inspection, or by guess and check, with and without the aid of a calculator.

Knowledge and Understanding
(Facts and Procedures)

Find the value of y in $2y + 7 = 21$

Explain your reasoning.

Knowledge and Understanding
(Conceptual Understanding)

Without solving, explain how you know the value for a and x in the following equations will be the same.

$$3x - 4 = 13$$

$$13 = 3a - 4$$

Reasoning and Proving
(Reflecting)

A local restaurant charges a $10 reservation fee and $25 per person for group parties.

Jeremiah used the equation below to determine the number of people at a graduation party that cost a total of $610.

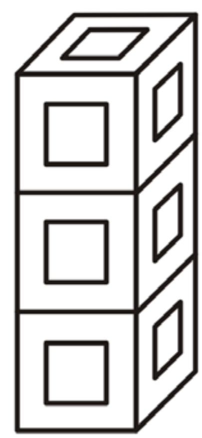
$25n + 10 = 610$, where n represents the number of people

First he tried $n = 20$. The value of the left side of the equation was 510. Next he tried $n = 10$.

Did Jeremiah demonstrate good reasoning in his second try? Explain your answer.

Problem Solving
(Representing, Connecting)

Each floor of a tall office building has one large window on each side and a skylight on the top floor. How many floors does the building have if there are 73 windows to clean?



Hint: Use interlocking cubes to help you.

Show your work.

Developing Proficiency

Grade 8

Name:
Date:

Expectation – Patterning and Algebra, 8m63: Solve and verify linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a "balance" model

Knowledge and Understanding
(Facts of Procedures)

Find the value of y in $21 + 2y = 237$.

Show all your work.

Knowledge and Understanding
(Conceptual Understanding)

Without solving, explain how you know the value for n and a in the following equations will be the same.

$3n – 2 = 7$

$6a – 4 = 14$

Problem Solving
(Reasoning and Proving)

Add any four numbers in a "box" on a calendar and I can tell you the location of the box.

	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Example:
Tell me the sum is 60, and I know the box begins at 11.
I used an equation to get the answer. What equation did I use?
(a) $11 + 4n = 60$ (b) $4n + 16 = 60$

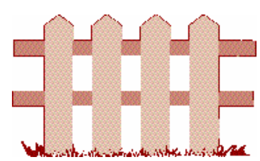
Give reasons for your answer.

Problem Solving
(Connecting, Representing)

There are 70.8 m of fencing around a rectangular field.
The length of the field is 20.4 m.
Determine the width of the field.

Show your work.

Hint: $2l + 2w = 70.8$



Developing Proficiency

Grade 9 Applied

Name:
Date:

Expectation – Number Sense and Algebra, NA2.07: Solve first-degree equations with nonfractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies)

Knowledge and Understanding
(Facts and Procedures)

Solve the given equation.

$$-10m + 8 = -5m - 8$$

Show your work.

Knowledge and Understanding
(Conceptual Understanding)

Maria's solution to $4a + 8 = 2a - 12$ appears below:

$$\begin{aligned} 4a + 8 &= 2a - 12 & \\ 2a + 8 &= -12 & \text{Step 1} \\ 2a &= -20 & \text{Step 2} \\ a &= -10 & \text{Step 3} \end{aligned}$$

Explain the reasoning in each step and how she can verify her answer.

Problem Solving
(Reflecting, Reasoning and Proving)

Farrell says that without solving the two equations given below he knows they have the same solution.

$$3n - 2 = 7$$

$$4 = 6a - 14$$

Do you agree or disagree? List reasons for your answer.

Problem Solving
(Connecting, Selecting Tools and Computational Strategies)

Bargain Bs determines the selling price of T-shirts by doubling its cost.

Ali bought a T-shirt from Bargain Bs for \$12.76. Determine Bargain Bs' cost for the T-shirt by using an equation.

Developing Proficiency

Grade 10 Applied

Name:
Date:

Expectation – Manipulating and Solving Algebraic Expressions, ML1.01: Solve first-degree equations involving one variable, including equations with fractional coefficients (e.g., using the balance analogy, computer algebra systems, paper and pencil)

Knowledge and Understanding
(Facts and Procedures)

Solve the given equation.

$$\frac{2x}{3}+7=2x+15$$

Show your work.

Knowledge and Understanding
(Conceptual Understanding)

A solution to $4a+9=\frac{a}{2}-12$ appears below:

$8a+18=a-24$ Step 1
$7a=-42$ Step 2
$a=-6$ Step 3

Explain the reasoning in each step and how to verify the answer.

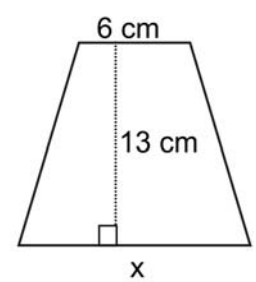
Problem Solving
(Reasoning and Proving, Reflecting)

Create a multi-step (at least two steps) equation that has $x = -4$ as its solution.

Verify the solution is $x = -4$ by solving your equation.

Problem Solving
(Connecting, Representing)

A trapezoid has an area of 91 cm^2. If the height is 13 cm and one side is 6 cm, find the length of the other side.



Problem Solving Across the Grades — Sample 1

Name:
Date:

Find 2 different ways to determine the solution to $35x + 52 = 647$
Show your work.

1.	2.

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1. Establishing a lower bound first, then building up

Problem-Solving Strategies:
- Use guess and check
- Perform multiple trials
- Use logical reasoning

Trial value for x	Value of $35x + 52$	Thinking
More than 10 by inspection	Mentally, $350 + 52 = 402$	I need about $\frac{1}{2}$ as much again to increase 402 to 647, and the 52 is not relatively that large, so try 15 for x.
15	$35 \times 15 + 52 = 577$	15 is a bit too small, and 647 is more than 35 more than 577, so try 17 for x.
17	$35 \times 17 + 52 = 647$	The solution to $35x + 52 = 647$ is $x = 17$.

2. Establishing an upper bound first, then moving down

Problem-Solving Strategies:
- Use guess and check
- Perform multiple trials
- Use logical reasoning

Trial value for x	Value of $35x + 52$	Thinking
Less than 20 by inspection	Mentally, $35 \times 20 = 700$	I need less than 20 for x, but not too much less since each time I reduce x by 1, the value of $35x + 52$ will decrease by 35.
18	$35 \times 18 + 52 = 682$	18 gives a value that is a bit too big, but only about 35 too big, so decrease x by 1.
17	$35 \times 17 + 52 = 647$	The solution to $35x + 52 = 647$ is $x = 17$.

3. Systematically testing the middle value between upper and lower bounds.

Problem-Solving Strategies:
- Use guess and check
- Perform multiple trials
- Use logical reasoning

Trial value for x	Value of $35x + 52$	Thinking
10	Mentally, $35 \times 10 = 350$ which will be too low, even after 52 more is added	I need more than 10 for x.
20	Mentally, $35 \times 20 = 700$ is already too big, even before adding 52	I need less than 20 for x, so x is between 10 and 20.
15	$35 \times 15 + 52 = 577$	15 is a bit too small, and 647 is more than 35 more than 577, so try 17 for x.
17	$35 \times 17 + 52 = 647$	The solution to $35x + 52 = 647$ is $x = 17$

4. Analysing numbers and their relationships deductively

Problem-Solving Strategies:
- Use deductive reasoning

- If I know that the solution to the equation is a whole number, then I need a multiple of 35 that is 52 less than 647, or $647 - 52 = 595$.
- Since 595 has 5 as its units digit, the value for x must be odd, i.e., I need an odd multiple of 35 to give 595.
- I can see that 35 divides into 595 somewhere between 10 and 20 times. $35\overline{)595}^{\,1?}$
- I can complete the long division or try, 35×13, 35×15, 35×17, and 35×19 until I get 595.
- Since $35 \times 17 = 595$ and $595 + 52 = 647$, therefore, $x = 17$ is the solution to $35x + 52 = 647$

Grades 8, 9, or 10

Students' solutions could include any of the Grade 7 answers.

Problem-Solving Strategies:
- Use algebraic reasoning

1. Use the balance approach

$$35x + 52 = 647$$

$$35x + 52 - 52 = 647 - 52 \quad \text{Undo 52 from both sides.}$$

$$35x = 595$$

$$\frac{35x}{35} = \frac{595}{35} \quad \text{Undo multiplication by 35 by dividing both sides by 35.}$$

$x = 17$ Students can verify by substituting $x = 17$ into the original equation.

Grade 10 Applied

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Problem-Solving Strategies:
- Use algebraic reasoning

2.

$$35x + 52 = 647$$

$$x + \frac{52}{35} = \frac{647}{35} \quad \text{Divide each term by 35}$$

$$x + \frac{52}{35} - \frac{52}{35} = \frac{647}{35} - \frac{52}{35} \quad \text{Subtract } \frac{52}{35} \text{ from each side}$$

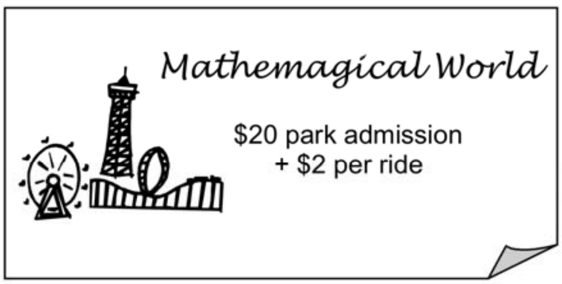
$$x = \frac{595}{35}$$

$$x = 17$$

Name:
Date:

Mathemagical World charges a $20 admission fee to the park plus an additional $2 for each ride. Radical Island charges $3 for each ride but does not charge an admission fee to the park.

Which amusement park costs less? *Show your work.*



Hint: Would this park cost less for every customer?

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1. Construct a table that shows the relationship between the cost of a day at the park and the number of rides for each amusement park.

Problem-Solving Strategies:
- Create a numerical model
- Use logical reasoning

Mathemagical World

Number of Rides	Cost ($)
0	20.00
5	30.00
10	40.00
15	50.00
20	60.00
25	70.00
30	80.00
n	$20 + 2n$

Radical Island

Number of Rides	Cost ($)
0	0
5	15.00
10	30.00
15	45.00
20	60.00
25	75.00
30	90.00
n	$3n$

If a customer goes on 20 rides, the cost is the same ($60) at both parks.
If a customer goes on fewer than 20 rides, it costs less to go to Radical Island.
If a customer goes on more than 20 rides, it costs less to go to Mathemagical World.

Grade 9

Students' solutions could include any of the Grades 7 and 8 answers.

Problem-Solving Strategies:
- Create a graphical model
- Use logical reasoning
- Use algebra

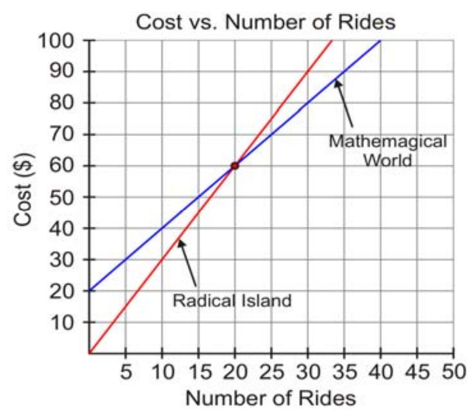
1. Students in Grade 9 math learn to solve linear systems graphically and interpret meaning of the point of intersection in the context of a problem.

Constructing a Graph Using Initial Value and Rate of Change

Determine the point of intersection of the two lines (20,60).
If a customer goes on 20 rides, the cost is the same ($60) at both parks.

If a customer goes on fewer than 20 rides, it costs less to go to Radical Island.

If a customer goes on more than 20 rides, it costs less to go to Mathemagical World.



2.

Money to Start With	Number of Rides at Mathemagical World	Number of Rides Radical Island
100	$100-20=80$ $\frac{80}{2}=40$ *40 rides* *(better deal)*	$\frac{100}{3}=33$ *33 rides*
50	$50-20=30$ $\frac{30}{2}=15$ *15 rides*	$\frac{50}{3}=16$ *16 rides* *(better deal)*
70	$70-20=50$ $\frac{50}{2}=25$ *25 rides* *(better deal)*	$\frac{70}{3}=23$ *23 rides*
60	$60-20=40$ $\frac{40}{2}=20$ *20 rides* *(same)*	$\frac{60}{3}=20$ *20 rides* *(same)*

Problem-Solving Strategies:
- Make an organized list
- Work backwards
- Look for a pattern

Both parks offer the same number of rides for $60.00.
If you go on fewer than 20 rides or have less than $60.00 to spend, go the Radical Island.
If you go on more than 20 rides, go to Mathemagical World.

3. Students in Grade 9 solve linear equations.

Students could recognize from the table of values that $20 + 2n = 3n$. Solving shows $n = 20$. Students then relate this solution to the context of the problem. (See conclusion for Grade 10 solution.)

Problem-Solving Strategies:
- Create an algebraic model
- Use logical reasoning

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Students in Grade 10 Applied Math learn to solve linear systems algebraically.

1. Solve a Linear System Algebraically

Problem-Solving Strategies:
- Create an algebraic model
- Use substitution/elimination

Write an equation that represents the cost of a day at each park.

Mathemagical World:	$C = 20 + 2n$	(1)
Radical Island:	$C = 3n$	(2)

Substitute C from equation (2) into C of equation (1) and solve for n.

$$C = 20 + 2n$$
$$3n = 20 + 2n$$
$$n = 20$$

Substitute $n = 20$ into equation (2) and solve for C

$$C = 3n$$
$$C = 3(20)$$
$$C = 60$$

Therefore, if a customer goes on 20 rides, the cost is the same ($60) at both parks.

Conclusion

As Radical Island has no admission fee, it initially costs less for a customer to go to this park.

Explore the cost, if a customer goes on fewer than 20 rides. Substitute any number less than 20 into both equations to determine which costs less:

Let $n = 19$.

For Mathemagical World,

$$\begin{aligned} C &= 20 + 2n \\ &= 20 + 2(19) \\ &= 58 \end{aligned}$$

For Radical Island

$$\begin{aligned} C &= 3n \\ &= 3(19) \\ &= 57 \end{aligned}$$

Therefore, it costs less to go to Radical Island when going on fewer than 20 rides.

Explore the cost, if a customer goes on 20 or more rides. Substitute any number more than 20 into both equations to determine which costs less:

Let $n = 21$.

For Mathemagical World,

$$\begin{aligned} C &= 20 + 2(21) \\ &= 20 + 42 \\ &= 62 \end{aligned}$$

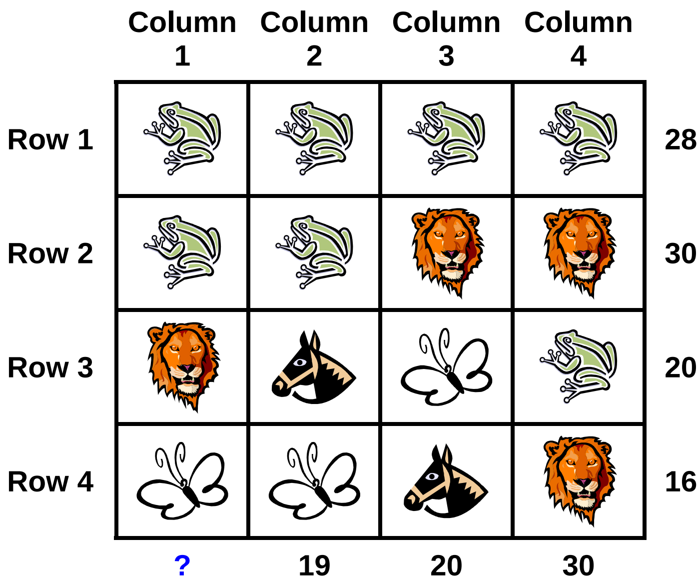
For Radical Island

$$\begin{aligned} C &= 3n \\ &= 3(21) \\ &= 63 \end{aligned}$$

Therefore, it costs less to go to Mathemagical World when going on more than 20 rides.

Name:
Date:

The numbers alongside each row and at the bottom of each column are the total of the values of the symbols within each row and column. If each symbol represents a whole number, find two different ways of determining the total value of the symbols in the first column.



1.	2.

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

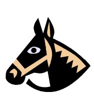
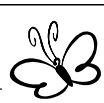
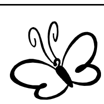
1. Students could determine the value of each symbol by inspection and relate their solution entirely in words.

Problem-Solving Strategies:
- Use logical reasoning

Conclusion	Reasoning
 = 7	From the first row four frogs total 28, therefore one frog must be 7.
 = 8	In the second row, the two frogs total 14. Subtracting fourteen from 30 means that two lions must total 16. Therefore, one lion is 8.
 +  = 5	From the third row, we subtract the lion and frog from 20, leaving us with a combined total for the horse and butterfly of 5.
 = 3	From the fourth row, we subtract the lion and the horse-butterfly combination from 16, leaving us with the butterfly of 3.
The total of the desired column is 25.	Two frogs make 14, one lion is 8, and one butterfly is 3.

2. Students could use variables to represent the various symbols in the problem and use guess and check or the balance model to determine the values.

Problem-Solving Strategies:
- Use logical reasoning
- Create a mathematical model
- Use guess and check

Conclusion	Reasoning
 = 7	$4f = 28$ Therefore, $f = 7$.
 = 8	$2f + 2l = 30$ (since $f = 7$) $14 + 2l = 30$ Therefore, $l = 8$
 +  = 5	$l + h + b + f = 20$ (since $f = 7$ and $l = 8$) $8 + h + b + 7 = 20$ Therefore, $h + b = 5$
 = 3	$b + b + h + l = 16$ $b + 5 + 8 = 16$ Therefore, b = 3
The total of the desired column is 25.	$2f + l + b$ $= 2(7) + 8 + 3 = 25$

3. The overall total for the columns must be equal to the overall totals for the rows. Find the difference to determine the missing column.

> **Problem-Solving Strategies:**
> - Look for a pattern

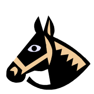
$$
\begin{aligned}
\textit{Row Total} &= \textit{Column Total}\\
28+30+20+16 &= ?+19+20+30\\
94 &= ?+69\\
? &= 94-69\\
? &= 25
\end{aligned}
$$

Grade 9

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Students may offer a more algebraic approach to solving the equations that model each row or column.

> **Problem-Solving Strategies:**
> - Make a mathematical model
> - Use algebra

Conclusion	Reasoning
 $=7$	$\begin{aligned} 4f &= 28\\ \frac{4f}{4} &= \frac{28}{4}\\ 7 &= f \end{aligned}$
 $=8$	$\begin{aligned} 2f+2l &= 30 \quad (\text{since } f=7)\\ 14+2l &= 30\\ 14+2l-14 &= 30-14\\ 2l &= 16\\ \frac{2l}{2} &= \frac{16}{2}\\ l &= 8 \end{aligned}$
 +  $=5$	$\begin{aligned} l+h+b+f &= 20 \quad (\text{since } f=7 \text{ and } l=8)\\ 8+h+b+7 &= 20\\ h+b+15-15 &= 20-15\\ h+b &= 5 \end{aligned}$
 $=3$	$\begin{aligned} b+b+h+l &= 16\\ b+5+8 &= 16\\ b+13 &= 16\\ b+13-13 &= 16-13\\ b &= 3 \end{aligned}$
The total of the desired column is 25.	$\begin{aligned} T &= 2f+l+b\\ &= 2(7)+8+3\\ T &= 25 \end{aligned}$

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Students may solve the remaining two equations in two unknowns using a system of linear equations.

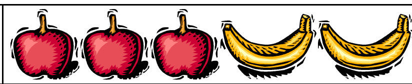
Problem-Solving Strategies:
- Make an algebraic model

<table>
<tr><th>Conclusion</th><th>Reasoning</th></tr>
<tr><td> = 7</td><td>$$4f = 28$$ $$\frac{4f}{4} = \frac{28}{4}$$ $$7 = f$$</td></tr>
<tr><td> = 8</td><td>$$2f + 2l = 30 \quad (\text{since } f = 7)$$ $$14 + 2l = 30$$ $$14 + 2l - 14 = 30 - 14$$ $$2l = 16$$ $$\frac{2l}{2} = \frac{16}{2}$$ $$l = 8$$</td></tr>
<tr><td> +  = 5</td><td>$$l + h + b + f = 20 \quad (\text{since } f = 7 \text{ and } l = 8)$$ $$8 + h + b + 7 = 20$$ $$h + b + 15 - 15 = 20 - 15$$ $$h + b = 5$$</td></tr>
<tr><td> = 3</td><td>$$b + b + h + l = 16$$ $$2b + h + 8 = 16$$ $$2b + h + 8 = 16 - 8$$ $$2b + h = 8$$</td></tr>
<tr><td colspan="2">Use either elimination or substitution to solve for b and h.</td></tr>
<tr><td colspan="2">$$\begin{array}{r} 2b + h = 8 \\ \underline{1b + h = 5} \\ b = 3 \end{array} \quad (\text{subtract the equations}) \qquad \begin{array}{r} b + h = 5 \\ 3 + b = 5 \\ h = 2 \end{array}$$</td></tr>
<tr><td>The total of the desired column is 25.</td><td>$$T = 2f + l + b$$ $$T = 2(7) + 8 + 3$$ $$T = 25$$</td></tr>
</table>

Problem Solving Across the Grades — Sample 3

Name:
Date:

Apples and bananas are sold in the market on the planet Linear. If the value of an apple and banana are whole numbers, find two different ways of determining the value of each.

Basket 1		= 36
Basket 2		= 51

1.	**2.**

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1. Students use the first equation to generate possible pairs of values for the apple and banana. They can then test each of the possible solutions in the second equation until they find one that provides the correct amount.

Problem-Solving Strategies:
- Use guess and check
- Make an organized list
- Look for a pattern

Note: Some students may see the pattern in the third column when they start checking and use that to hypothesize the correct solution.

From basket 1, 3 apples and 2 bananas equal 36		Check in second basket: 4 apples and 3 bananas equal 51	True or False
Possible apple value	**Possible banana value**		
0	18	$4(0)+3(18)$ $=0+54$ $=54$	False
1	16.5	Not possible since answer must be whole.	
2	15	$4(2)+3(15)$ $=8+45$ $=53$	False
3	13.5	Not possible since answer must be whole.	
4	12	$4(4)+3(12)$ $=16+36$ $=52$	False
5	10.5	Not possible since answer must be whole.	
6	9	$4(6)+3(9)$ $=24+27$ $=51$	True
7	7.5		
8	6		
9	4.5		
10	3		
11	1.5		
12	0		

The value of an apple is 6 and a banana is 9.

2. Similar to previous method, students find the difference between the baskets and use this new relationship to generate a list of possible pairs of values for the apple and banana. They can then test each of the possible solutions in either equation until they find one that provides the correct amount.

Problem-Solving Strategies:
- Use guess and check
- Make an organized list
- Look for a pattern.

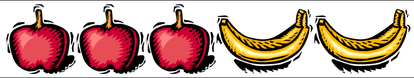
Difference between Basket 1 and Basket 2: one apple and one banana is 15.

Note: Some students may see the pattern in the third column when they start checking and use that to hypothesize the correct solution.

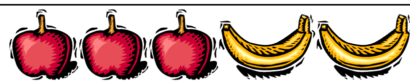
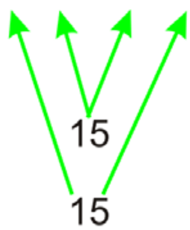
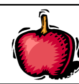
Possible apple value	Possible banana value	Check in first basket: 3 apples and 2 bananas equals 36	True or False
0	15	$3(0)+2(15)$ $=0+30$ $=30$	False
1	14	$3(1)+2(14)$ $=3+28$ $=31$	False
2	13	$3(2)+2(13)$ $=6+26$ $=32$	False
3	12	$3(3)+2(12)$ $=9+24$ $=33$	False
4	11	$3(4)+2(11)$ $=12+22$ $=34$	False
5	10	$3(5)+2(10)$ $=15+20$ $=35$	False
6	9	$3(6)+2(9)$ $=18+18$ $=36$	True
7	8		
8	7		
9	6		
10	5		
11	4		
12	3		
13	2		
14	1		
15	0		

Problem-Solving Strategies:
- Use logical reasoning
- Draw a picture

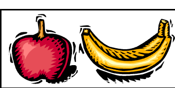
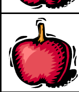
3. Step 1: The difference between Baskets 1 and 2 is 1 apple and 1 banana. The difference in the value between Baskets 1 and 2 is 15. Therefore, 1 apple plus 1 banana is 15.

Basket 1		= 36
Basket 2		= 51
Difference		= 15

Step 2: Apply the results of Step 1 to basket 1, leaving 1 apple valued at 6.

Basket 1		= 36
Pairing what we know		= 30
Remaining		= 6

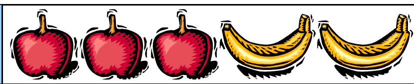
Step 3: Apply the results of Step 1 and Step 2, leaving 1 banana valued at 9.

We know		= 15
and		= 6
therefore		= 9

Students' solutions could include any of the Grades 7 and 8 answers.

3. Students could isolate for one of the two unknowns and graph the equations either by hand or using technology.

Problem-Solving Strategies:
- Create a mathematical model
- Draw a diagram (graph)

Basket 1		= 36	$3a + 2b = 36$
Basket 2		= 51	$4a + 3b = 51$

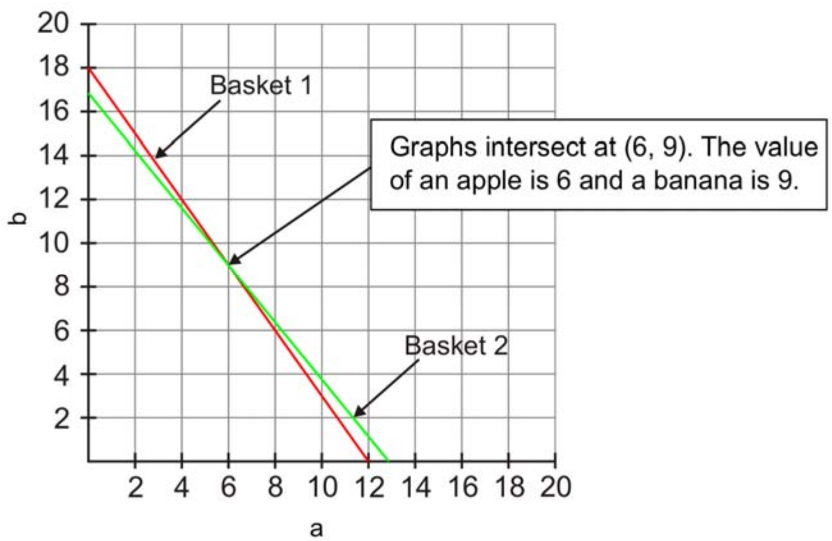
Basket 1: $3a + 2b = 36$

Isolating for b, we get: $b = -\frac{3}{2}a + 18$

Basket 2: $4a + 3b = 51$

Isolating for b, we get: $b = -\frac{4}{3}a + 17$

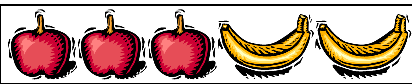
Graph equations from Basket 1 and Basket 2.



Students' solutions could include any of the Grades 7, 8, and 9 answers.

4. Students can model the equations algebraically and solve a system of linear equations using either elimination or substitution.

Problem-Solving Strategies:
- Create a mathematical model
- Use algebra

Basket 1		= 36	$3a + 2b = 36$
Basket 2		= 51	$4a + 3b = 51$

Solution:

$3a + 2b = 36$ (Multiply equation by 4)
$4a + 3b = 51$ (Multiply equation by 3)

$$\begin{array}{r} 12a + 8b = 144 \\ \underline{12a + 9b = 153} \\ -b = -9 \\ b = 9 \end{array}$$
(Subtract equations)

Substitute $b = 9$ into first equation, $3a + 2b = 36$

$$\begin{aligned} 3a + 2(9) &= 36 \\ 3a + 18 &= 36 \\ 3a &= 18 \\ a &= 6 \end{aligned}$$

The value of an apple is 6 and a banana is 9.

Is This Always True?

(Reflecting, Reasoning and Proving)

Grades 7–10

Name:
Date:

1. Myrna declares that 3x = 2x + x is true for x = 1, and x = 2, and x = 3.

   Is 3x = 2x + x always true for any values of x?

2. Irene says that if you subtract fractions of the form $\frac{n}{n+1} - \frac{n-1}{n}$,

   you always get a numerator of 1 (for example, $\frac{5}{6} - \frac{4}{5}, \frac{6}{7} - \frac{5}{6}$).

   Is this true for all natural numbers?

Grades 7 and 8

Sample Solutions

Problem-Solving Strategies:
- Use algebraic reasoning
- Use concrete materials

1. **Yes.**

   a. Students could build a pattern with toothpicks.

Term	Pattern
▲	3(1)
▲▲	3(2)
▲▲▲	3(3)
	⋮
	3(x)

same as

Term	Pattern
▲	2(1) + 1
▲▲	2(2) + 2
▲▲▲	2(3) + 3
	⋮
	2(x) + x

b. Since the expressions on the left side and right side of the equation are equivalent, the values of the left and right sides will be equivalent for any value of x.
(Students could demonstrate this using algebra tiles).

Problem-Solving Strategies:
- Use algebraic reasoning
- Use concrete materials

2. **Yes.** The fractions look like:

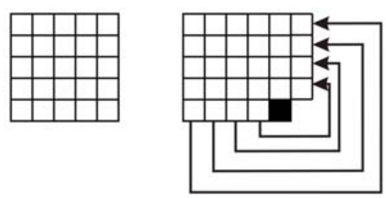
$$\frac{2}{3}-\frac{1}{2}=\frac{4-3}{6}=\frac{1}{6}$$
$$\frac{3}{4}-\frac{2}{3}=\frac{9-8}{12}=\frac{1}{12}$$
$$\frac{4}{5}-\frac{3}{4}=\frac{16-15}{20}=\frac{1}{20}$$

Problem-Solving Strategies:
- Look for a pattern
- Draw a diagram
- Use algebra
- Use inductive reasoning

I can see that the pattern for the final numerator is always 1. It comes from the square of a number minus the product of the whole numbers 1 above and 1 below it, e.g., $5^2 - 4 \times 6 = 1$, $6^2 - 5 \times 7 = 1$, $7^2 - 6 \times 8 = 1$.

The result is always 1 since we are comparing the area of a square to the area of a rectangle 1 unit shorter and 1 unit wider.

For example:



In Grade 10, students can follow this teacher-assisted proof:

Problem-Solving Strategies:
- Use algebra

The expression $\frac{n}{n+1}-\frac{n-1}{n}$ could be simplified.

$$\frac{n}{n+1} - \frac{n-1}{n}$$

$$=\frac{n^2}{n(n+1)} - \frac{(n+1)(n-1)}{n(n+1)}$$ Determine a common denominator.

$$=\frac{n^2-(n^2-1)}{n-(n+1)}$$

$$=\frac{n^2-n^2+1}{n(n+1)}$$ Expand and simplify.

$$=\frac{1}{n(n+1)}$$ When n is a natural number, the numerator will always be 1.

Is This Always True?
(Reflecting, Reasoning and Proving)

Grades 9 and 10

Name:
Date:

1. Trevor says that if you multiply each term in an equation by 2, the solution to the equation will remain the same.

   For example:

   $5x - 12 = 3$ has $x = 3$ as its solution and $2\ (5x) - 2\ (12) = 2\ (3)$ or $10x - 24 = 6$ also has $x = 3$ as its solution.

   Is this true for all equations? Explain.

2. Sami claims that if each term in an equation is increased by 2, the solution will remain the same.

   For example:

   $2x = 2$ has $x = 1$ as its solution. $2x + 2 = 2 + 2$, or $2x + 2 = 4$, or $2x = 2$ also has $x = 1$ as its solution.

   Is this true for all equations? Explain.

3. Kaye says that every time you move a number from one side of an equation to the other side, you change its sign.

   Is this always true? Explain.

1. **Yes.**

   a. Multiplying each term by 2 doubles each side of the equation and preserves the balance.

Problem-Solving Strategies:
- Use logical reasoning

   b. Students can use concrete materials such as algebra tiles to demonstrate that doubling each term doubles each side and maintains the balance.

Problem-Solving Strategies:
- Use concrete materials

   c. In Grade 10, students graph using intercepts. Students can graph an equation and graph other equations whose terms are multiples of the original. They will recognize the graphs are the same line, thus they must be the same equation.

   For example: $x+2y=6$

   $2x+4y=12$

   $3x+6y=18$

Problem-Solving Strategies:
- Use a graph

   d. Start with an easy problem: $x=5$

   $2x=10$ Double each term.
   $x=5$

   $3x=15$ Triple each term.
   $x=3$

   $4x=20$ Multiply each term.
   $x=5$

   It appears the answer remains the same.

Problem-Solving Strategies:
- Solve and easier problem
- Look for a pattern
- Use deductive reasoning

   Next consider a harder problem. $x+1=5$
   $x=4$

   $2x+2=10$ Double each term and check if $x=4$.
   $x=4$

   $3x+3=15$ Triple each term and check if $x=4$.
   $x=4$

   $4x+4=20$ Multiply each term by 4 and check if $x=4$.
   $x=4$

   Students could consider increasing complex equations and deduce that it is always true.

2. **No.** In fact this is rarely true.
   To keep an equation balanced, an equal amount must be added or subtracted to the whole side of the equation, not to each term.

   One counter-example would be: $3x + 4 = 10$ has $x = 2$ as its solution.
   If we add 2 to each term, we get $(3x + 2) + (4 + 2) = 10 + 2$,
   or $3x + 8 = 12$, $3x = 4$, so $x = \frac{4}{3}$ is its solution, not $x = 2$.

Problem-Solving Strategies:
- Find a counter-example

3. **No.** Consider the counter-example: When solving $5x = 35$, we get $x = \frac{35}{5}$ or $x = 7$. The sign on 5 did not change as we "undid" the multiplication by 5 using division by 5. It is only when "undoing" addition and subtraction that the sign appears to change.

Problem-Solving Strategies:
- Use algebraic reasoning
- Use a counter-example